

# Can Effective Field Theory of inflation generate large tensor-to-scalar ratio within Randall Sundrum single braneworld?

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In this paper my prime objective is to explain the generation of large tensor-to-scalar ratio from the single field sub-Planckian inflationary paradigm within Randall Sundrum (RS) single braneworld scenario in a model independent fashion. By explicit computation I have shown that the effective field theory prescription of brane inflation within RS single brane setup is consistent with sub-Planckian excursion of the inflaton field, which will further generate large value of tensor-to-scalar ratio, provided the energy density for inflaton degrees of freedom is high enough compared to the brane tension in high energy regime. Finally, I have mentioned the stringent theoretical constraint on positive brane tension, cut-off of the quantum gravity scale and bulk cosmological constant to get sub-Planckian field excursion along with large tensor-to-scalar ratio as recently observed by BICEP2 or at least generates the tensor-to-scalar ratio consistent with the upper bound of Planck (2013 and 2015) data and Planck+BICEP2+Keck Array joint constraint.

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## I. INTRODUCTION

It is a very good-old assumption from superstring theory [1–3] that we are living in 11 dimensions and different string field theoretic setups are connected with each other via stringy duality conditions. Among varieties of string theories, the 10-dimensional  $E_8 \otimes E_8$  heterotic string theory is a strong candidate for our real world as the theory may contain the standard model of particle physics and is related to an 11-dimensional theory written on the orbifold  $\mathbf{R}^{10} \otimes \mathbf{S}^1/Z_2$ . Within this field theoretic setup, the standard model particle species are confined to the 4-dimensional space-time which is the sub-manifold of  $\mathbf{R}^4 \otimes \mathbf{S}^1/Z_2$ . On the contrary, the graviton degrees of freedom propagate in the total space-time. In a most simplified situation, one can think about a 5-dimensional problem where the

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matter fields are confined to the 4-dimensional spacetime while gravity acts in 5 dimensional bulk spacetime [4, 5]. Amongst very successful propositions for extra dimensional models, Randall & Sundrum (RS) one brane [6] and two brane [7] models are very famous theoretical prescription in which our observable universe is embedded on 3-brane which is exactly identical to a domain wall in the context of 5-dimensional anti-de Sitter ( $\text{AdS}_5$ ) space-time. Various cosmo-phenomenological consequences along with inflation have been studied from RS setup in refs. [8–19, 21, 22].

The primordial inflation has two key predictions - creating the scalar density perturbations and the tensor perturbations during the accelerated phase of expansion [23, 24]. Very recently, BICEP2 <sup>1</sup> [25] team reported the detection of the primordial tensor perturbations through the B-mode polarization as:

$$r = 0.20^{+0.07}_{-0.05} \text{ (within } 2\sigma \text{ C.L.)}, \quad (1.1)$$

where  $r$  is the tensor-scalar ratio. Explaining this large tensor-to-scalar ratio is a challenging issue for particle cosmologist because of the Lyth bound [32], one would expect a super-Planckian excursion <sup>2</sup> of the inflaton field in order to generate large tensor-to-scalar ratio. It is important to mention here that super-Planckian field excursion computed from the inflationary paradigm is necessarily required to embed the setup with effective field theory description <sup>3</sup>. At present it is a very challenging task for the theoretical physicists to propose a new mechanism or technique through which it is possible to accommodate sub-Planckian inflation to generate large tensor-to-scalar ratio. The

<sup>1</sup> BICEP2 result was quite recently put into question by several works [26–29]. Also accounting for the contribution of foreground dust will shift the value of tensor-to-scalar ratio  $r$  downward by an amount which will be better constrained by the joint analysis performed by Planck and BICEP2/Keck Array team [30]. The final result is expressed as a likelihood curve for  $r$ , and yields an upper limit  $r_{0.05} < 0.12$  at  $2\sigma$  confidence. Marginalizing over dust and  $r$ , lensing B-modes are detected at  $7\sigma$  significance. Very recently in [31] the Planck team also fixed the upper bound on the tensor-to-scalar ratio is  $r_{0.002} < 0.11$  at  $2\sigma$  C.L. and perfectly consistent with the joint analysis performed by Planck and BICEP2/Keck Array team.

<sup>2</sup> Field excursion of the inflation field is defined as:  $\Delta\phi = \phi_{cmb} - \phi_e$ , where  $\phi_{cmb}$  represent the field value of the inflaton at the momentum scale  $k$  which satisfies the equality,  $k = aH = -\eta^{-1} \approx k_*$ , where  $(a, H, \eta)$  represent the scale factor, Hubble parameter, the conformal time and pivot momentum scale respectively. Also  $\phi_e$  is the field value of the inflaton defined at the end of inflation. Here the super-Planckian excursion is described by,  $|\Delta\phi| > M_p$ , which is applicable for large field models of inflation [33–37] and sub-Planckian excursion is characterized by,  $|\Delta\phi| < M_p$ , which hold good in case of small field models of inflation [38–41].

<sup>3</sup> In case of super-Planckian field excursion it is necessarily required to introduce the higher order quantum corrections including the effect of higher derivative interactions appearing through the local modifications to GR plays significant role in this context [11]. For an example, within 4D Effective Field Theory picture incorporating the local corrections in GR one can write the action as,

$$S_{local} = \int d^4x \sqrt{-(4)g} \left[ \sum_{n=1}^{\infty} \mathbf{a}_n {}^{(4)}R^n + \sum_{m=1}^{\infty} \mathbf{b}_m \left( {}^{(4)}R_{\mu\nu} {}^{(4)}R^{\mu\nu} \right)^m + \sum_{p=1}^{\infty} \mathbf{c}_p \left( {}^{(4)}R_{\alpha\beta\delta\eta} {}^{(4)}R^{\alpha\beta\delta\eta} \right)^p \right].$$

In this case the appropriate choice of the co-efficients  $\mathbf{a}_n, \mathbf{b}_m, \mathbf{c}_p$  of the correction factors would modify the UV behaviour of gravity. But such local modification of the renormalizable version of GR typically contain debris like massive ghosts which cannot be regularized or avoided using any field theoretic prescriptions. If the quantum correction to the usual classical theory of gravity represented via Einstein-Hilbert term is dominated by higher derivative nonlocal corrections [42–44] then one can avoid such ghost degrees of freedom, as the role of these corrections are significant in super-Planckian (or trans-Planckian) scale to make the theory UV complete [42]. For an example, within 4D Effective Field Theory picture incorporating the non-local corrections in the gravity sector one can write the action as [43]:

$$S_{non-local} = \int d^4x \sqrt{-(4)g} \left[ RF_1(\square)R + R_{\mu\nu}F_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}F_3(\square)R^{\mu\nu\alpha\beta} + RF_4(\square)\nabla_\mu\nabla_\nu\nabla_\gamma\nabla_\eta R^{\mu\nu\gamma\eta} \right. \\ \left. + R^{\nu_1\rho_1\alpha_1}F_5(\square)\nabla_{\rho_1}\nabla_{\alpha_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\alpha R^{\mu\nu\rho\alpha} + R^{\mu_1\nu_1\rho_1\alpha_1}F_6(\square)\nabla_{\rho_1}\nabla_{\alpha_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\alpha R^{\mu\nu\rho\alpha} \right]$$

where  $F_i(\square)\forall i$  are analytic entire functions containing higher derivatives up to infinite order, where  $\square = g^{\mu\nu}\nabla_\mu\nabla_\nu$  is the 4D d'Alembertian operator. On the other hand in the matter sector incorporating the effects of quantum correction through the interaction between heavy and light (inflaton) field sector and finally integrating out the heavy degrees of freedom from the 4D Effective Field Theory picture the matter action, which admits a systematic expansion within the light inflaton sector can be written as [34, 45]:

$$S_{matter}[\phi, \Psi] = \int d^4x \sqrt{-(4)g} \left[ \mathcal{L}_{inf}[\phi] + \mathcal{L}_{heavy}[\Psi] + \mathcal{L}_{int}[\phi, \Psi] \right] \xrightarrow{\text{Remove } \Psi} e^{iS_{matter}[\phi]} = \int [\mathcal{D}\Psi] e^{iS_{matter}[\phi, \Psi]} \\ S_{matter}[\phi] = \int d^4x \sqrt{-(4)g} \left[ \mathcal{L}_{inf}[\phi] + \sum_{\alpha} J_{\alpha}(g) \frac{\mathcal{O}_{\alpha}[\phi]}{M_p^{\Delta_{\alpha}-4}} \right]$$

where  $J_{\alpha}(g)$  are dimensionless Wilson coefficients that depend on the couplings  $g$  of the UV theory, and  $\mathcal{O}_{\alpha}[\phi]$  are local operators of dimension  $\Delta_{\alpha}$ . This procedure typically generates all possible effective operators  $\mathcal{O}_{\alpha}[\phi]$  consistent with the symmetries of the UV theory. Also  $\mathcal{L}_{inf}[\phi]$  and  $\mathcal{L}_{heavy}[\Psi]$  describe the part of total Lagrangian density  $\mathcal{L}$  involving only the light and heavy fields, and  $\mathcal{L}_{int}[\phi, \Psi]$  includes all possible interactions involving both sets of fields within Effective Field Theory prescription. After removal of heavy degrees of freedom the effective action is splitted into a renormalizable part:

$$\mathcal{L}_{inf}[\phi] = \frac{g^{\mu\nu}}{2} (\partial_{\mu}\phi)(\partial_{\nu}\phi) - V_{ren}(\phi)$$

first possibility of addressing this issue is to incorporate the features of spectral tilt, running and running of the running by modifying the scale invariant power spectrum. Obviously, the current data can also be explained by the sub-Planckian excursion of the inflaton field in the context of single field inflation as discussed in [46–50], where in these class of models sufficient amount of running and running of the running in tensor-to-scalar ratio has been taken care of. A small class of potentials inspired from particle physics phenomenology i.e. high scale models of inflation in the context of MSSM,  $\text{MSSM} \otimes U(1)_{B-L}$  etc [51–54] will serve this purpose. The next possibility is modified gravity or beyond General Relativistic (GR) framework through which it is possible to address this crucial issue within single field inflationary scenario where the effective field theory description holds perfectly. The prime motivation of this work to show explicitly how one can address this issue in beyond GR prescription. In this work I investigate the possibility for RS single brane setup in which one can generate large tensor-to-scalar ratio along with sub-Planckian field excursion from a large class of models of inflation within effective field theory prescription [33, 34, 45, 55–63], and within this setup it is feasible to describe a system through the lowest dimension operators compatible with the underlying symmetries <sup>4</sup>.

In this paper, I derive the direct connection between field excursion and tensor-to-scalar ratio in the context of effective theory inflation within Randall-Sundrum (RS) braneworld scenario in a model independent fashion. For clarity in the present context the bulk space-time is assumed to have 5 dimensions. By explicit computation I have shown that the effective field theory of brane inflation within RS setup is consistent with sub-Planckian VEV and field excursion, which will further generate large value of tensor-to-scalar ratio when the energy density for inflaton degrees of freedom is high enough as compared to the visible and hidden brane tensions in high energy regime. Last but not the least, I have mentioned the stringent constraint condition on positive brane tension as well as on the cut-off of the quantum gravity scale to get sub-Planckian field excursion along with large tensor-to-scalar ratio.

## II. BRANE INFLATION WITHIN RANDALL-SUNDRUM SINGLE BRANE SETUP

Let me start the discussion with a very brief introduction to RS single brane setup. The RS single brane setup and its generalized version from a Minkowski brane to a Friedmann- Robertson-Walker (FRW) brane were derived as solutions in specific choice of coordinates of the 5D Einstein equations in the bulk, along with the junction conditions which are applied at the  $\mathbf{Z}_2$ -symmetric single brane. A broader perspective, with non-compact dimensions, can be obtained via the well known covariant Shiromizu-Maeda-Sasaki approach [79], in which the brane and bulk metrics take its generalized structure. The key point is to use the Gauss-Codazzi equations to project the 5D bulk curvature along the brane using the covariant formalism. Here I start with the well known 5D Randall Sundrum (RS) single brane model action given by [6]:

$$S_{RS} = \int d^5x \sqrt{-(^{(5)}g)} \left[ \frac{M_5^3}{2} ({}^{(5)}R - 2\Lambda_5 + \mathcal{L}_{bulk} + (\mathcal{L}_{brane} - \sigma) \delta(y) \right], \quad (2.1)$$

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and a sum of non-renormalizable corrections appearing through the operators  $\mathcal{O}_\alpha[\phi]$ . Such operators of dimensions less than four are called “relevant operators”. They dominate in the IR and become small in the UV. In 4D Effective Field Theory the operators of dimensions greater than four are called irrelevant operators. These operators become small in the IR regime, but dominate in the UV end. However such corrections are extremely hard to compute and at the same time the theoretical origin of all such corrections is not at all clear till now as it completely belongs to the hidden sector of the theory [45]. One of the possibilities of the origin of such hidden sector heavy field is higher dimensional Superstring Theory or its low energy supergravity version. Such a higher dimension setups dimensionally reduced to the 4D Effective Field Theory version via various compactifications. In such a case the corrections arising from graviton loops will always be weighted by the UV cut-off scale  $\Lambda_{UV}$  which is fixed at Planck scale  $M_p$ , while those coming from heavy sector fields will be suppressed by the background scale of heavy physics relevant for those fields  $M_s$ , where  $M_s < \Lambda_{UV} \approx M_p$ . Present observational status suggests that the scale of such hidden scale is constrained around the GUT scale ( $10^{16}$  GeV) [47, 48]. In this connection Randall-Sundrum (RS) model is one of possible remedies to solve the trans-Planckian problem of field excursion as the 5D cut-off scale of such theory (see section II for details) is one order smaller than the 4D cut-off scale of the Effective Field Theory i.e. the Planck scale  $M_p$  to explain the latest ATLAS bound on the lightest graviton mass and the Higgs mass within the estimated  $\sim 125$  GeV against large radiative correction upto the cut-off of the Model [15] in the phenomenological ground. In this work using model independent semi-analytical analysis within inflationary setup we have explicitly shown that 5D cut-off  $M_5$  of RS model is also one order smaller than the 4D cut-off scale  $M_p$  (see section III for details). This also suggests that within RS setup the higher order quantum corrections appearing in the gravity as well in the matter sector of the theory is very small in the 4D Effective Field Theory version. During our analysis we have further taking an ansatz where the non-renormalizable 4D Planck scale suppressed effective operators only modify the effective potential. Consequently with the renormalizable part of the potential  $V_{ren}$  such corrections will add and finally give rise to the total potential  $V(\phi)$  as stated in Eq (2.12).

<sup>4</sup> Assisted inflation [64–70] and N-flation [71–73] within multi-field inflationary description, asymptotically free gravity [42–44, 74–76], shift symmetry [77, 78] are the various possibilities in which it is possible to achieve sub-Planckian field excursion along with large tensor-to-scalar ratio and finally the trans-Planckian field excursion issue can be resolved within Effective Field Theory prescription.

where the extra dimension “y” is non-compact for which the covariant formalism is applicable. Here  $M_5$  be the 5D quantum gravity cut-off scale,  $\Lambda_5$  be the 5D bulk cosmological constant,  $\mathcal{L}_{bulk}$  be the bulk field Lagrangian density,  $\mathcal{L}_{brane}$  signifies the Lagrangian density for the brane field contents. It is important to mention that the scalar inflaton degrees of freedom is embedded on the 3 brane which has a positive brane tension  $\sigma$  and it is localized at the position of orbifold point  $y = 0$  in case of single brane. The 5D field equations in the bulk, including explicitly the contribution of the RS single brane is given by [4, 79]:

$${}^{(5)}G_{AB} = \frac{1}{M_5^3} \left[ -\Lambda_5 {}^{(5)}g_{AB} + {}^{(5)}T_{AB} + T_{\mu\nu}^{\text{brane}} \delta_A^\mu \delta_B^\nu \delta(y) \right] \quad (2.2)$$

where  ${}^{(5)}T_{AB}$  characterizes any 5D energy-momentum tensor of the gravitational sector within bulk spacetime. On the other hand the total energy-momentum tensor on the brane is given by:  $T_{\mu\nu}^{\text{brane}} = T_{\mu\nu} - \sigma g_{\mu\nu}$ , where  $T_{\mu\nu}$  is the energy-momentum tensor of particles and fields confined to the single brane. Further applying the well known Israel–Darmois junction conditions at the brane [4, 79] finally one can arrive at the 4-dimensional Einstein induced field equations on the single brane given by [4, 5, 79]:

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{1}{M_p^2} T_{\mu\nu} + \left( \frac{8\pi}{M_5^3} \right)^2 \mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu} , \quad (2.3)$$

where  $T_{\mu\nu}$  represents the energy-momentum on the single brane,  $\mathcal{S}_{\mu\nu}$  is a rank-2 tensor that contains contributions that are quadratic in the energy momentum tensor  $T_{\mu\nu}$  [4, 79] and  $\mathcal{E}_{\mu\nu}$  characterizes the projection of the 5-dimensional Weyl tensor on the 3-brane and physically equivalent to the non-local contributions to the pressure and energy flux for a perfect fluid [4, 5, 79].

In a cosmological framework, where the 3-brane resembles our universe and the metric projected onto the brane is an homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) metric, the Friedmann equation becomes [4, 5, 79]:

$$H^2 = \frac{\Lambda_4}{3} + \frac{\rho}{3M_p^2} + \left( \frac{4\pi}{3M_5^3} \right)^2 \rho^2 + \frac{\epsilon}{a^4} , \quad (2.4)$$

where  $\epsilon$  is an integration constant. The four and five-dimensional cosmological constants are related by [4, 5, 79]:

$$\Lambda_4 = \frac{4\pi}{M_5^3} \left( \Lambda_5 + \frac{4\pi}{3M_5^3} \sigma^2 \right) , \quad (2.5)$$

where  $\sigma$  is the 3-brane tension. Within RS setup the quantum gravity cut-off scale i.e. the 5D Planck mass and effective 4D Planck mass are connected through the visible brane tension as:

$$M_5^3 = \sqrt{\frac{4\pi\sigma}{3}} M_p. \quad (2.6)$$

Assuming that, as required by observations, the 4D cosmological constant is negligible  $\Lambda_4 \approx 0$  in the early universe the localized visible brane tension is given by:

$$\sigma = \sqrt{-\frac{3}{4\pi} M_5^3 \Lambda_5} = \sqrt{-24 M_5^3 \tilde{\Lambda}_5} > 0 \quad (2.7)$$

where  $\tilde{\Lambda}_5$  be the scaled 5D bulk cosmological constant defined as:

$$\tilde{\Lambda}_5 = \frac{\Lambda_5}{32\pi} < 0. \quad (2.8)$$

Also the last term in Eq. (2.4) rapidly becomes redundant after inflation sets in, the Friedmann equation in RS braneworld becomes [4, 5, 79]:

$$H^2 = \frac{\rho}{3M_p^2} \left( 1 + \frac{\rho}{2\sigma} \right) \quad (2.9)$$

where  $\sigma$  be the positive brane tension,  $\rho$  signifies the energy density of the inflaton field  $\phi$  and  $M_p = 2.43 \times 10^{18}$  GeV be the reduced 4D Planck mass. Using Eq (2.7) in Eq (2.6), the 5D quantum gravity cut-off scale can be expressed in terms of 5D cosmological constant as:

$$M_5^3 = \sqrt[3]{-\frac{4\pi\Lambda_5}{3}} M_p^{4/3} = \sqrt[3]{-\frac{128\pi^2\tilde{\Lambda}_5}{3}} M_p^{4/3}. \quad (2.10)$$

In the low energy limit  $\rho \ll \sigma$  in which standard GR framework can be retrieved. On the other hand, in the high energy regime  $\rho \gg \sigma$  as the effect of braneworld correction factor is dominant which is my present focus in this paper. Consequently in high energy limit  $\rho \gg \sigma$ , Eq (2.9) is written using the slow-roll approximation as:

$$H^2 \approx \frac{\rho^2}{6M_p^2\sigma} \approx \frac{V^2(\phi)}{6M_p^2\sigma}, \quad (2.11)$$

where  $V(\phi)$  be the inflaton single field potential which is expanded in a Taylor series around an intermediate field value  $\phi_i < \phi_0 (< M_p) < \phi_e$ <sup>5</sup> as:

$$\begin{aligned} V(\phi) &= V(\phi_0) + V'(\phi_0)(\phi - \phi_0) + \frac{V''(\phi_0)}{2}(\phi - \phi_0)^2 + \frac{V'''(\phi_0)}{6}(\phi - \phi_0)^3 + \frac{V''''(\phi_0)}{24}(\phi - \phi_0)^4 + \dots, \\ &= \sum_{n=0}^{\infty} \frac{V'^n(\phi_0)}{n!}(\phi - \phi_0)^n, \end{aligned} \quad (2.12)$$

where  $V(\phi_0) \ll M_p^4$  denotes the height of the potential, and the coefficients:  $V'(\phi_0) \leq M_p^3$ ,  $V''(\phi_0) \leq M_p^2$ ,  $V'''(\phi_0) \leq M_p$ ,  $V''''(\phi_0) \leq \mathcal{O}(1)$ , determine the shape of the potential in terms of the model parameters. The *prime* denotes the derivative w.r.t.  $\phi$ . Here as a special case one can consider a situation where the intermediate field value  $\phi_0$  is identified with the VEV of the inflaton field  $\phi$  i.e.

$$\langle 0|\phi|0\rangle = \phi_0, \quad (2.13)$$

where  $|0\rangle$  be the Bunch-Davies vacuum state using which the VEV is computed in curved space-time. In a most simplest case the numerical value of the VEV is computed from the flatness condition:

$$V'(\phi_0) = 0 \quad (2.14)$$

provided  $V''(\phi_0) > 0$ . In a more advanced situation where inflation is driven by *saddle point* and *inflection point*, one can impose the flatness constraint on the potential as:

$$V'(\phi_0) = 0 = V''(\phi_0) \quad (2.15)$$

for *saddle point* [54, 80] and

$$V''(\phi_0) = 0 \quad (2.16)$$

for *inflection point* [51–53, 81]<sup>6</sup>. Moreover here it is important mention that the inflaton field belongs to the visible sector of RS setup in which effective field theory prescription perfectly holds good. Even for zero VEV of the inflaton,  $\langle 0|\phi|0\rangle = \phi_0 = 0$ , Eq (2.12) also holds good. One can further simplify the expression for the potential by applying  $\mathbf{Z}_2$  symmetry in the inflaton field as:

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \lambda' M_p^{-2}\phi^6 + \lambda'' M_p^{-4}\phi^8 + \dots = \sum_{m=0}^{\infty} \mathbf{C}_{2m}\phi^{2m}. \quad (2.17)$$

<sup>5</sup> Here  $\phi_i$  and  $\phi_e$  represent the inflaton field value at the starting point of inflation and at the end of inflation.

<sup>6</sup> The present observational data from Planck and BICEP2 prefers the *inflection point* models of inflation compared to the *saddle point*, as the predicted value for the scalar spectral tilt obtained from *saddle point* inflationary models is low.

where the expansion co-efficients are defined as:

$$\mathbf{C}_0 = V_0, \quad (2.18)$$

$$\mathbf{C}_2 = m^2 = V''(0), \quad (2.19)$$

$$\mathbf{C}_4 = \lambda = \frac{V''''(0)}{4!}, \quad (2.20)$$

$$\mathbf{C}_6 = \lambda' = \frac{M_p^2 V'''''(0)}{6!}, \quad (2.21)$$

$$\mathbf{C}_8 = \lambda'' = \frac{M_p^4 V''''''(0)}{8!}. \quad (2.22)$$

Within high energy limit the slow-roll parameters in the visible brane can be expressed as [4, 8, 10]:

$$\epsilon_b(\phi) \approx \frac{2M_p^2 \sigma(V'(\phi))^2}{V^3(\phi)}, \quad (2.23)$$

$$\eta_b(\phi) \approx \frac{2M_p^2 \sigma V''(\phi)}{V^2(\phi)}, \quad (2.24)$$

$$\xi_b^2(\phi) \approx \frac{4M_p^4 \sigma^2 V'(\phi) V'''(\phi)}{V^4(\phi)}, \quad (2.25)$$

$$\sigma_b^3(\phi) \approx \frac{8M_p^6 \sigma^3 (V'(\phi))^2 V''''(\phi)}{V^4(\phi)}. \quad (2.26)$$

and consequently the number of e-foldings can be written as [4, 8, 10]:

$$\Delta N_b = |N_b(\phi_{cmb}) - N_b(\phi_e)| \approx \frac{1}{2\sigma M_p^2} \int_{\phi_e}^{\phi_{cmb}} d\phi \frac{V^2(\phi)}{V'(\phi)} \quad (2.27)$$

where  $\phi_e$  corresponds to the field value at the end of inflation which can be obtained from the following equation:

$$\max_{\phi=\phi_e} [\epsilon_b, |\eta_b|, |\xi_b^2|, |\sigma_b^3|] = 1. \quad (2.28)$$

In terms of the momentum, the number of e-foldings,  $N_b(k)$ , can be expressed as [82]:

$$N_b(k) \approx 71.21 - \ln\left(\frac{k}{k_*}\right) + \frac{1}{4} \ln\left(\frac{V_*}{M_P^4}\right) + \frac{1}{4} \ln\left(\frac{V_*}{\rho_e}\right) + \frac{1-3w_{int}}{12(1+w_{int})} \ln\left(\frac{\rho_{rh}}{\rho_e}\right), \quad (2.29)$$

where  $\rho_e$  is the energy density at the end of inflation,  $\rho_{rh}$  is an energy scale during reheating,  $k_* = a_* H_*$  is the present Hubble scale,  $V_*$  corresponds to the potential energy when the relevant modes left the Hubble patch during inflation corresponding to the momentum scale  $k_*$ , and  $w_{int}$  characterises the effective equation of state parameter between the end of inflation, and the energy scale during reheating. Within the momentum interval,  $k_e < k < k_{cmb}$ , the corresponding number of e-foldings is given by,  $\Delta N_b$ , as

$$\Delta N_b = |N_b(k_e) - N_b(k_{cmb})| = \ln\left(\frac{k_{cmb}}{k_e}\right) = \ln\left(\frac{a_{cmb}}{a_e}\right) + \ln\left(\frac{H_{cmb}}{H_e}\right) = \ln\left(\frac{a_{cmb}}{a_e}\right) + \ln\left(\frac{V_{cmb}}{V_e}\right) \quad (2.30)$$

where  $(a_{cmb}, H_{cmb})$  and  $(a_e H_e)$  represent the scale factor and the Hubble parameter at the CMB scale and end of inflation. One can estimate the contribution of the last term of the right hand side by using Eq (2.12) as:

$$\begin{aligned} \left(\frac{V_{cmb}}{V_e}\right) &= \left[1 + \sum_{n=1}^{\infty} \frac{V'^n(\phi_0)}{n!V(\phi_0)} (\phi_{cmb} - \phi_0)^n\right] \left[1 + \sum_{j=1}^{\infty} \frac{V'^j(\phi_0)}{j!V(\phi_0)} (\phi_e - \phi_0)^j\right]^{-1}, \\ &\approx \left[1 + \sum_{n=1}^{\infty} \frac{V'^n(\phi_0)}{n!V(\phi_0)} (\phi_{cmb} - \phi_0)^n - \sum_{j=1}^{\infty} \frac{V'^j(\phi_0)}{j!V(\phi_0)} (\phi_e - \phi_0)^j - \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{V'^n(\phi_0) V'^j(\phi_0)}{n!j!V^2(\phi_0)} (\phi_{cmb} - \phi_0)^n (\phi_e - \phi_0)^j\right], \\ &\approx [1 + W - Q], \end{aligned} \quad (2.31)$$

where  $W$  and  $Q$  represent two series sum given by:

$$W = \sum_{j=1}^{\infty} \frac{1}{(j-1)!} \left( \frac{\Delta\phi}{M_p} \right) \frac{V'^j(\phi_0) M_p^j}{V(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{j-1}, \quad (2.32)$$

$$Q = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{V'^n(\phi_0) V'^j(\phi_0) M_p^{n+j}}{V^2(\phi_0)} \left\{ \frac{1}{n!j!} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{n+j} + \frac{1}{(n-1)!j!} \left( \frac{\Delta\phi}{M_p} \right) \left( \frac{\phi_e - \phi_0}{M_p} \right)^{n+j-1} \right\} \quad (2.33)$$

where the field excursion is defined as,  $\Delta\phi = \phi_{cmb} - \phi_e$ , where  $\phi_{cmb}$  and  $\phi_e$  signify the inflaton field value at the at the last scattering surface (LSS) of CMB or more precisely at the horizon crossing<sup>7</sup> and at the end of inflation respectively. Now I explicitly show that both of the series sum are convergent in the present context. To hold the effective field theory prescription one need to satisfy the following sets of criteria:

- (1).  $\left( \frac{\phi_e - \phi_0}{M_p} \right) \leq 1$ ,
- (2).  $\left( \frac{\Delta\phi}{M_p} \right) \leq 1$ ,
- (3).  $\frac{V'^j(\phi_0) M_p^j}{V(\phi_0)} \leq 1 \forall j$ ,
- (4).  $\frac{V'^n(\phi_0) V'^j(\phi_0) M_p^{n+j}}{V^2(\phi_0)} \leq 1 \forall (n, j)$ .

This implies that, both  $W < 1$  and  $Q < 1$  are convergent and from Eq (2.35) we get:

$$\left( \frac{V_{cmb}}{V_e} \right) \approx 1, \quad (2.34)$$

which perfectly holds good for zero VEV inflaton case. Let us investigate the  $\mathbf{Z}_2$  symmetric case in which one can write:

$$\begin{aligned} \left( \frac{V_{cmb}}{V_e} \right) &= \left[ 1 + \sum_{n=1}^{\infty} \frac{C_{2n}}{V_0} \phi_{cmb}^{2n} \right] \left[ 1 + \sum_{j=1}^{\infty} \frac{C_{2j}}{V_0} \phi_e^{2j} \right]^{-1}, \\ &\approx \left[ 1 + \sum_{n=1}^{\infty} \frac{C_{2n}}{V_0} \phi_{cmb}^{2n} - \sum_{j=1}^{\infty} \frac{C_{2j}}{V_0} \phi_e^{2j} - \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{C_{2n} C_{2j}}{V_0^2} \phi_{cmb}^{2n} \phi_e^{2j} \right], \\ &\approx [1 + W_0 - Q_0], \end{aligned} \quad (2.35)$$

where  $W_0$  and  $Q_0$  represent two series sum given by:

$$W_0 = 2 \sum_{j=1}^{\infty} j \left( \frac{\Delta\phi}{M_p} \right) \frac{C_{2j} M_p^{2j}}{V_0} \left( \frac{\phi_e}{M_p} \right)^{2j-1}, \quad (2.36)$$

$$Q_0 = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} \frac{C_{2n} C_{2j} M_p^{2(n+j)}}{V_0^2} \left\{ \left( \frac{\phi_e}{M_p} \right)^{2(n+j)} + 2n \left( \frac{\Delta\phi}{M_p} \right) \left( \frac{\phi_e}{M_p} \right)^{2(n+j)-1} \right\} \quad (2.37)$$

Here also the similar criteria hold good to apply the effective field theory prescription which make the series sum  $W_0$  and  $Q_0$  convergent. Consequently, for all the physical situations described in this paper Eq (2.30) reduces to:

$$\Delta N_b \approx \ln \left( \frac{k_{cmb}}{k_e} \right) = \ln \left( \frac{a_{cmb}}{a_e} \right). \quad (2.38)$$

---

<sup>7</sup> Here horizon crossing stands for the physical situation where the corresponding momentum scale satisfies the equality  $k = \frac{2\pi}{\lambda_w} = aH$ , where  $\lambda_w$  be the associated wavelength of the scalar and tensor modes whose snapshot are observed at the LSS of CMB. After crossing the horizon all such modes goes to the super-Hubble region in which the momentum scale  $k \gg aH$  i.e.  $\lambda_w \ll \frac{2\pi}{aH}$ , which implies the corresponding wavelengths of the scalar and tensor modes are too small to be detected. On the other hand, before the horizon crossing there will be region in a smooth patch within sub-Hubble region where the corresponding momentum scale  $k \ll aH$  i.e.  $\lambda_w \gg \frac{2\pi}{aH}$ , which can be detected via various observational probes.



### III. FIELD EXCURSION WITHIN EFFECTIVE THEORY DESCRIPTION

In the high energy limit of RS braneworld the tensor-to-scalar ratio satisfies the following consistency condition at the leading order of the effective field theory:

$$r_b(k) = \frac{P_T(k)}{P_S(k)} = 24\epsilon_b = \frac{48M_p^2\sigma(V'(\phi))^2}{V^3(\phi)} \quad (3.1)$$

where  $P_S(k)$  and  $P_T(k)$  are the scalar and tensor power spectrum at any scale  $k$ . It is important to note that the following operator relationship holds good in the high energy limit of RS braneworld:

$$\frac{d}{d\phi} = -\frac{V^2}{2\sigma M_p^2 V'} \frac{d}{d \ln k}. \quad (3.2)$$

In Eq (3.1) the tensor-to-scalar ratio can be parametrized at any arbitrary momentum scale as:

$$r_b(k) = \begin{cases} r_b(k_*) & \text{for Case I} \\ r_b(k_*) \left(\frac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1} & \text{for Case II} \\ r_b(k_*) \left(\frac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1+\frac{\alpha_T(k_*)-\alpha_S(k_*)}{2!} \ln\left(\frac{k}{k_*}\right)} & \text{for Case III} \\ r_b(k_*) \left(\frac{k}{k_*}\right)^{n_T(k_*)-n_S(k_*)+1+\frac{\alpha_T(k_*)-\alpha_S(k_*)}{2!} \ln\left(\frac{k}{k_*}\right)+\frac{\kappa_T(k_*)-\kappa_S(k_*)}{3!} \ln^2\left(\frac{k}{k_*}\right)} & \text{for Case IV.} \end{cases} \quad (3.3)$$

where  $k_*$  be the pivot scale of momentum. In Eq (3.3) the subscript  $(T, S)$  signifies the tensor and scalar modes obtained from cosmological perturbation in RS braneworld. Here  $(n_T, n_S)$ ,  $(\alpha_T, \alpha_S)$  and  $(\kappa_T, \kappa_S)$  represent the tensor and scalar spectral tilt, running and running of the running in RS braneworld respectively. See appendix where all these definitions are explicitly given. Also in Eq (3.3) I mention four possibilities as given by:

- **Case I** stands for a situation where the spectrum is scale invariant,
- **Case II** stands for a situation where spectrum follows power law feature through the spectral tilt  $(n_S, n_T)$ ,
- **Case III** signifies a situation where the spectrum shows deviation from power law in presence of running of the spectral tilt  $(\alpha_S, \alpha_T)$  along with logarithmic correction in the momentum scale (as appearing in the exponent) and
- **Case IV** characterizes a physical situation in which the spectrum is further modified compared to the **Case III**, by allowing running of the running of spectral tilt  $(\kappa_S, \kappa_T)$  along with square of the momentum dependent logarithmic correction.

Further combining Eq (3.1) and Eq (3.2) together and performing the momentum as well as the slow-roll integration I get:

$$\frac{1}{2}\sqrt{\frac{\sigma}{3}} \left| \int_{k_e}^{k_{cmb}} d \ln k \sqrt{r_b(k)} \right| = \frac{1}{M_p} \left| \int_{\phi_e}^{\phi_{cmb}} d\phi \sqrt{V(\phi)} \right|. \quad (3.4)$$

Finally substituting Eq (4.11) and Eq (4.15) on Eq (3.4) I get:



$$\left| \frac{\Delta\phi}{M_p} \right| = \frac{1}{2} \sqrt{\frac{\sigma}{3\mathcal{V}_{inf}}} \times \begin{cases} \sqrt{r_b(k_*)} \frac{|\Delta N_b|}{2\sqrt{r_b(k_*)}} & \text{for Case I} \\ \frac{n_T(k_*) - n_S(k_*) + 1}{n_T(k_*) - n_S(k_*) + 1} \left| 1 - e^{-\Delta N_b \left( \frac{n_T(k_*) - n_S(k_*) + 1}{2} \right)} \right| & \text{for Case II} \\ \sqrt{r_b(k_*)} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{2(\alpha_T(k_*) - \alpha_S(k_*))}} \sqrt{\frac{2\pi}{(\alpha_T(k_*) - \alpha_S(k_*))}} & \\ \left| \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} \right) \right. & \text{for Case III} \\ \quad \left. - \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} - \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \Delta N_b \right) \right| & \\ \sqrt{r_b(k_*)} \left| \left( \frac{3}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} \right. \right. & \\ \quad \left. \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \{1 - e^{-\Delta N_b}\} - \left( \frac{1}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} \right. \right. & \\ \quad \left. \left. + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \Delta N_b e^{-\Delta N_b} \right. & \\ \quad \left. - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) (\Delta N_b)^2 e^{-\Delta N_b} \right. & \\ \quad \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^3 e^{-\Delta N_b} \right| & \text{for Case IV.} \end{cases} \quad (3.5)$$

Here all the observables appearing in the left side of Eq (3.5) can also be expressed in terms of the slow-roll parameters in RS single braneworld. See the appendix for details. Further using the the limiting results on  $\Delta N_b$  I get:

$$\lim_{\Delta N_b \rightarrow \text{small}} \left| \frac{\Delta\phi}{M_p} \right| = \frac{1}{2} \sqrt{\frac{\sigma}{3\mathcal{V}_{inf}}} \times \begin{cases} \sqrt{r_b(k_*)} \frac{|\Delta N_b|}{2\sqrt{r_b(k_*)}} & \text{for Case II} \\ \sqrt{r_b(k_*)} \frac{|\Delta N_b|}{2\sqrt{r_b(k_*)}} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{2(\alpha_T(k_*) - \alpha_S(k_*))}} & \text{for Case III} \\ \sqrt{r_b(k_*)} \frac{|\Delta N_b|}{2\sqrt{r_b(k_*)}} \left| 1 - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) \Delta N_b \right. & \\ \quad \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^2 \right| & \text{for Case IV.} \end{cases} \quad (3.6)$$

Most importantly Eq (4.20) and Eq (4.21) fix the value of  $\Delta N_b$  within the desired range demanded by the observational probes. This can be easily done by putting constraint on the brane tension of the single brane and the Taylor expansion co-efficients of the effective potential within RS setup. Also this makes the analysis consistent presented in this paper. Further from Eq (4.20) and Eq (4.21) one can write the field excursion for the both the physical situations as:

$$\underline{\text{Without } \mathbf{Z}_2} : \quad \left| \frac{2\sigma \Delta N_b V'(\phi_0) M_p}{V^2(\phi_0)} \right| \approx \left| \frac{\Delta\phi}{M_p} \right| \leq 1, \quad (3.7)$$

$$\underline{\text{With } \mathbf{Z}_2} : \quad \left| \frac{4\sigma \phi_e \Delta N_b m^2 M_p}{V_0^2} \right| \approx \left| \frac{\Delta\phi}{M_p} \right| \leq 1. \quad (3.8)$$

Now using Eq (3.7) and Eq (3.8) one can express the analytical bound on the positive brane tension  $\sigma$  as:

$$\underline{\text{Without } \mathbf{Z}_2} : \quad \sigma \leq \left| \frac{V^2(\phi_0)}{2\Delta N_b V'(\phi_0) M_p} \right|, \quad (3.9)$$

$$\underline{\text{With } \mathbf{Z}_2} : \quad \sigma \leq \left| \frac{V_0^2}{4\phi_e \Delta N_b m^2 M_p} \right|. \quad (3.10)$$

Now I will explicitly show the details of each of the constraints on  $\sigma$  computed from Eq (3.9) and Eq (3.10). To serve this purpose let me now first write down the Taylor expansion co-efficient of the generic potential

$$\begin{pmatrix} V(\phi_0) \\ V'(\phi_0) \\ V''(\phi_0) \\ V'''(\phi_0) \\ V''''(\phi_0) \\ \vdots \\ \vdots \end{pmatrix} \approx \begin{pmatrix} V(\phi_*) \\ V'(\phi_*) \\ V''(\phi_*) \\ V'''(\phi_*) \\ V''''(\phi_*) \\ \vdots \\ \vdots \end{pmatrix}. \quad (3.15)$$

Now in case of  $\mathbf{Z}_2$  symmetric situation with zero VEV one can rewrite the solution of matrix equation as:

$$\begin{pmatrix} V_0 \\ m^2 \\ 24\lambda \\ \dots \\ \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} 1 & -\phi_* & \frac{\phi_*^2}{2} & -\frac{\phi_*^3}{6} & \frac{\phi_*^4}{24} & \dots & \dots \\ 0 & 0 & 1 & -\phi_* & \frac{\phi_*^2}{2} & \dots & \dots \\ 0 & 0 & 0 & 0 & 1 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} V(\phi_*) \\ V''(\phi_*) \\ V''''(\phi_*) \\ \dots \\ \dots \\ \dots \\ \dots \end{pmatrix}. \quad (3.16)$$

where

$$\phi_* \approx \phi_{cmb} = \left( \underbrace{\frac{\phi_e}{M_p}}_{\leq 1} + \underbrace{\frac{\Delta\phi}{M_p}}_{\leq 1} \right) M_p \leq M_p. \quad (3.17)$$

Further applying convergence criteria one can recast Eq (3.16) as:

$$\begin{pmatrix} V_0 \\ m^2 \\ 24\lambda \\ \dots \\ \dots \\ \dots \\ \dots \end{pmatrix} \approx \begin{pmatrix} V(\phi_*) \\ V''(\phi_*) \\ V''''(\phi_*) \\ \dots \\ \dots \\ \dots \\ \dots \end{pmatrix}. \quad (3.18)$$

The present analysis clearly shows that the scale of inflation is given by:

$$\sqrt[4]{\mathcal{V}_{inf}} \approx \sqrt[12]{2\pi^2 P_S(k_*) r(k_*)} M_p^{1/3} \sigma^{1/6} \lesssim \sqrt[4]{\frac{3}{2} P_S(k_*) r(k_*) \pi^2} M_p \quad (3.19)$$

Now using Eq (3.15) and Eq (3.18) along with Eq (3.19) here I get the following expression for the analytical bound on the positive brane tension  $\sigma$  in terms of inflationary observables in RS single braneworld setup as:

$$\textbf{Without } \mathbf{Z}_2 : \quad \sigma \leq \left| \frac{3456 P_S(k_*) \pi^2 M_p^4}{(\Delta N_b)^6 (r(k_*))^2} \right|, \quad (3.20)$$

$$\textbf{With } \mathbf{Z}_2 : \quad \sigma \leq \left| \frac{3\sqrt{3} P_S(k_*) r(k_*) \pi^2 M_p^4}{4\Delta N_b \left( n_S(k_*) - 1 + \frac{r(k_*)}{4} \right)} \right|. \quad (3.21)$$

where  $\phi_e \leq M_p$  have been used in Eq (3.21).

Further using Eq (3.20), Eq (3.21) and Eq (2.6) it is possible to write down the analytical expression for the upper bound of the 5D Planck mass in terms of 4D Planck mass and various inflationary observables as:

$$\textbf{Without } \mathbf{Z}_2 : \quad M_5 \leq \sqrt[6]{\left| \frac{2\sqrt{2} r(k_*) P_S(k_*) \pi^3}{\Delta N_b} \right|} M_p, \quad (3.22)$$

$$\textbf{With } \mathbf{Z}_2 : \quad M_5 \leq \sqrt[6]{\left| \frac{P_S(k_*) r(k_*) \pi^3}{\Delta N_b \left( n_S(k_*) - 1 + \frac{3r(k_*)}{8} \right)} \right|} M_p. \quad (3.23)$$

Finally using Eq (2.7), Eq (2.8) and Eq (3.20-3.23) it is possible to write down the analytical expression for the upper bound on the magnitude of 5D bulk cosmological constant in terms of 4D Planck mass and various inflationary observables as:

$$\textbf{Without } \mathbf{Z}_2 : \quad \tilde{\Lambda}_5 = \frac{\Lambda_5}{32\pi} \geq -\frac{9}{48} \sqrt{\left| \frac{(2r(k_*))^{3/2} P_S^3(k_*) \pi^5}{2(\Delta N_b)^3} \right|} M_p^5, \quad (3.24)$$

$$\text{With } \mathbf{Z}_2 : \quad \tilde{\Lambda}_5 = \frac{\Lambda_5}{32\pi} \geq -\frac{9}{384} \sqrt{\left| \frac{P_S^3(k_*) r^3(k_*) \pi^5}{(\Delta N_b)^3 \left( n_S(k_*) - 1 + \frac{3r(k_*)}{8} \right)^3} \right|} M_p^5. \quad (3.25)$$

Within Planck's observable region of  $\Delta N_b \sim \mathcal{O}(8-10)$ , it is possible to constrain the power spectrum:  $P_S$ , spectral tilt:  $n_S$ , running of the spectral tilt:  $\alpha_S$ , and running of running of the spectral tilt:  $\kappa_S$ , for **Planck+WMAP-9+high L+BICEP2** data sets [83, 84]:

$$0.15 \leq r_b(k_*) \leq 0.27 \quad (3.26)$$

$$\ln(10^{10} P_S) = 3.089_{-0.027}^{+0.024} \quad (\text{within } 2\sigma \text{ C.L.}), \quad (3.27)$$

$$n_S = 0.9600 \pm 0.0071 \quad (\text{within } 3\sigma \text{ C.L.}), \quad (3.28)$$

$$\alpha_S = dn_S/d\ln k = -0.022 \pm 0.010 \quad (\text{within } 1.5\sigma \text{ C.L.}), \quad (3.29)$$

$$\kappa_S = d^2 n_S/d\ln k^2 = 0.020_{-0.015}^{+0.016} \quad (\text{within } 1.5\sigma \text{ C.L.}). \quad (3.30)$$

and for **Planck+WMAP-9+high L** data sets [25]:

$$r_b(k_*) < 0.12 \quad (3.31)$$

$$\ln(10^{10} P_S) = 3.089_{-0.027}^{+0.024} \quad (\text{within } 2\sigma \text{ C.L.}), \quad (3.32)$$

$$n_S = 0.9603 \pm 0.0073 \quad (\text{within } 3\sigma \text{ C.L.}), \quad (3.33)$$

$$\alpha_S = dn_S/d\ln k = -0.013 \pm 0.009 \quad (\text{within } 1.5\sigma \text{ C.L.}), \quad (3.34)$$

$$\kappa_S = d^2 n_S/d\ln k^2 = 0.020_{-0.015}^{+0.016} \quad (\text{within } 1.5\sigma \text{ C.L.}). \quad (3.35)$$

which will fix the field excursion in a sub-Planckian region by putting required constraint on the positive brane tension  $\sigma$  as discussed earlier. Now using these combined constraints it is possible to estimate the approximated numerical bound of the various parameters- brane tension ( $\sigma$ ), 5D Planck mass ( $M_5$ ) and 5D cosmological constant ( $\tilde{\Lambda}_5$ ) lying within the following window <sup>8</sup>:

$$\text{Without } \mathbf{Z}_2 : \quad \sigma \leq \mathcal{O}(10^{-9}) M_p^4, \quad M_5 \leq \mathcal{O}(0.04) M_p, \quad \tilde{\Lambda}_5 \geq -\mathcal{O}(10^{-15}) M_p^5, \quad (3.36)$$

$$\text{With } \mathbf{Z}_2 : \quad \sigma \leq \mathcal{O}(10^{-9}) M_p^4, \quad M_5 \leq \mathcal{O}(0.05) M_p, \quad \tilde{\Lambda}_5 \geq -\mathcal{O}(10^{-15}) M_p^5. \quad (3.37)$$

Also I get the following bound on the suppression pre-factor as appearing in the right side of Eq (3.5):

$$\frac{1}{2} \sqrt{\frac{\sigma}{3\mathcal{V}_{inf}}} < \mathcal{O}(0.09 - 0.16). \quad (3.38)$$

Substituting all of these contributions stated in Eq (4.7-4.9) to Eq (3.5) and further using Eq (3.26,3.31,3.38) the upper bound of the field excursion ( $|\Delta\phi|$ ) is constrained within the following sub-Planckian regime <sup>9</sup>:

<sup>8</sup> In order to recover the observational successes of general relativity, the high-energy regime where significant deviations occur must take place before nucleosynthesis. Table-top tests of Newton's laws put the lower bound on the brane tension and 5D Planck scale as:  $\sigma > \mathcal{O}(2.86 \times 10^{-86}) M_p^4$  and  $M_5 > \mathcal{O}(4.11 \times 10^{-11}) M_p$ . But such lower bound will not be able to produce large tensor-to-scalar ratio as required by BICEP2 and the upper bound of Planck.

<sup>9</sup> In the case of single field models in four dimensions, assuming the monotonous behaviour of the slow-roll parameter during inflation it has been shown that the tensor-to-scalar ratio  $r > 0.1$  requires field excursions that are very very close to or above the Planck scale cut-off [85, 86], which is completely in agreement with the well known *Lyth bound* [32]. But if the slow-roll parameters follow non-monotonous behaviour during inflation [51-53, 87], then by modifying the power law parameterization of the primordial power spectrum in presence of running and running of the running of the running of spectral tilt of the power spectrum it is possible to generate tensor-to-scalar ratio  $r > 0.1$  from sub-Planckian field excursion within the framework of Effective Field Theory [47, 48]. In this work, I have explicitly shown that in context of Randall Sundrum single brane cosmological setup, by tuning the brane tension in the high density/ high energy regime it is possible to generate tensor-to-scalar ratio  $r > 0.1$ , provided the constrained value of field excursion is lesser compared to the result available in case of low density/ low energy regime in the braneworld and the model independent analysis validates the Effective

$$\begin{aligned}
\left| \frac{\Delta\phi}{M_p} \right| \leq & \underbrace{\frac{1}{2} \sqrt{\frac{\sigma}{3\mathcal{V}_{inf}}}}_{\mathcal{O}(0.09-0.16)} \times \underbrace{\begin{cases} \mathcal{O}(2.7-5.1) \\ \mathcal{O}(2.7-4.6) \\ \mathcal{O}(0.6-1.8) \\ \mathcal{O}(0.2-0.3) \end{cases}}_{\text{From low density regime of RS}} = \underbrace{\begin{cases} \mathcal{O}(0.24-0.81) & \text{for Case I} \\ \mathcal{O}(0.24-0.73) & \text{for Case II} \\ \mathcal{O}(0.05-0.28) & \text{for Case III} \\ \mathcal{O}(0.02-0.05) & \text{for Case IV} \end{cases}}_{\text{From high density regime of RS}} \quad (3.39)
\end{aligned}$$

which is consistent with all the observational constraints mentioned earlier. Now in the low energy regime when the energy density of inflaton  $\rho \ll \sigma$  then, in this limit, the suppression pre-factor turns out to be:

$$\lim_{\rho \ll \sigma} \left[ \frac{1}{2} \sqrt{\frac{\sigma}{3\mathcal{V}_{inf}}} \right] \rightarrow 1. \quad (3.40)$$

Using this limiting result it is possible to obtain also the relation between field excursion and tensor-to-scalar ratio from Eq (3.5) in case of usual GR prescribed effective field theory setup. For the details see the refs. [46–50] where such limit and their cosmological consequences are elaborately studied. Now let me concentrate on the **first case** of Eq (3.5), which is the most simplest physical situation. If I take the limit,  $\rho \ll \sigma$ , then it absolutely reduces to the good-old *Lyth bound* in which for  $\Delta N_b \sim \mathcal{O}(8-10)$  super-Planckian field excursion  $|\Delta\phi| \sim \mathcal{O}(2.7-5.1) M_p > M_p$  is required to generate large tensor-to-scalar ratio as observed by BICEP2 or at least generates the tensor-to-scalar ratio consistent with the upper bound of Planck. Now in the RS single braneworld setup by setting the brane tension in the above mentioned desired value and fixing the scale of inflation in the vicinity of GUT scale it is possible to generate large tensor-to-scalar ratio using sub-Planckian field excursion for which it is possible to describe the setup by using effective field theory of inflation. But only in the **last case** of Eq (3.5) in the limit  $\rho \ll \sigma$  it is possible to obtain sub-Planckian field excursion  $|\Delta\phi| \sim \mathcal{O}(0.2-0.3) M_p < M_p$  to get large value of tensor-to-scalar ratio [46–48]. If we now switch on the effect of single brane in RS setup then due to the presence of the suppression pre-factor as mentioned in Eq (3.38) the field excursion further reduces to the GUT scale i.e.  $|\Delta\phi| \sim \mathcal{O}(0.02-0.05) M_p < M_p$ .

#### IV. CONCLUSION

To summarize, in the present article, I have established a methodology for generating sub-Planckian field excursion along with large tensor-to-scalar ratio in a single brane RS braneworld scenario for generic model of inflation with and without  $\mathbf{Z}_2$  symmetry in the most generalized form of inflationary potential. I have investigated this scenario by incorporating various parametrization in the power spectrum for scalar and tensor modes as well as in the tensor-to-scalar ratio as required by the observational probes. Using the proposed technique I have further derived a analytical as well as the numerical constraints on the positive brane tension, 5D Planck scale and 5D bulk cosmological constant in terms of the 4D Planck scale. Finally, I have given an estimation of the field excursion which lies within a sub-Planckian regime and makes the embedding of inflationary paradigm in RS single braneworld via effective field theory prescription consistent.

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Field Theory prescription more compared the case discussed in [47, 48]. In the low density/ low energy regime Randall Sundrum single braneworld model exactly goes to the General Relativistic limit and hence it is possible to achieve the stringent bound derived in [47, 48]. On the other hand in the high density regime of the Randall Sundrum braneworld without modifying the power-law/scale invariant parametrization of the primordial power spectrum it is possible to achieve  $r > 0.1$  with sub-Planckian field excursion just by allowing fine tuning in the brane tension. But if we still modify the primordial power spectrum and allow the contributions from running and running of the running of the spectral tilt of the primordial power spectrum then we can probe lesser value of the field excursion compared to the General Relativistic limiting (low density) result derived in [47, 48]. Additionally in the high density regime of Randall Sundrum braneworld by allowing fine-tuning in the brane tension, it is possible to increase the upper bound of the energy scale of inflation and at best it is possible to achieve the upper bound of tensor-to-scalar ratio as observed by BICEP2 i.e.  $r \sim 0.27$  within the Effective Field Theoretic regime of inflation. See Eq (3.11) for details. However, the recent joint analysis performed by Planck mission along with BICEP2/Keck Array team [30] and Planck 2015 data [31] fix the upper bound of tensor-to-scalar ratio at  $r < 0.12$ , which can be surely achieved by the prescribed methodology established within the framework of Randall Sundrum single brane inflationary scenario.

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## Appendix

### A. Consistency relations in RS single braneworld

In the context of RS single braneworld the spectral indices  $(n_S, n_T)$ , running  $(\alpha_S, \alpha_T)$  and running of the running  $(\kappa_T, \kappa_S)$  at the momentum pivot scale  $k_*$  can be expressed as [4, 8, 10]:

$$n_S(k_*) - 1 = 2\eta_b(\phi_*) - 6\epsilon_b(k_*), \quad (4.1)$$

$$n_T(k_*) = -3\epsilon_b(k_*) = -\frac{r_b(k_*)}{8}, \quad (4.2)$$

$$\alpha_S(k_*) = 16\eta_b(k_*)\epsilon_b(k_*) - 18\epsilon_b^2(k_*) - 2\xi_b^2(k_*), \quad (4.3)$$

$$\alpha_T(k_*) = 6\eta_b(k_*)\epsilon_b(k_*) - 9\epsilon_b^2(k_*), \quad (4.4)$$

$$\kappa_S(k_*) = 152\eta_b(k_*)\epsilon_b^2(k_*) - 32\epsilon_b(k_*)\eta_b^2(k_*) - 108\epsilon_b^3(k_*) - 24\xi_b^2(k_*)\epsilon_b(k_*) + 2\eta_b(k_*)\xi_b^2(k_*) + 2\sigma_b^3(k_*), \quad (4.5)$$

$$\kappa_T(k_*) = 66\eta_b(k_*)\epsilon_b^2(k_*) - 12\epsilon_b(k_*)\eta_b^2(k_*) - 54\epsilon_b^3(k_*) - 6\epsilon_b(k_*)\xi_b^2(k_*). \quad (4.6)$$

Let me compute the following significant contributions which appeared at the left side of Eq (3.5) in terms of slow-roll parameters in RS single braneworld:

$$n_T(k_*) - n_S(k_*) + 1 = \left( \frac{d \ln r_b(k)}{d \ln k} \right)_* = \left[ \frac{r_b(k_*)}{8} - 2\eta_b(k_*) \right], \quad (4.7)$$

$$\alpha_T(k_*) - \alpha_S(k_*) = \left( \frac{d^2 \ln r_b(k)}{d \ln k^2} \right)_* = \left[ \left( \frac{r_b(k_*)}{8} \right)^2 - \frac{20}{3} \left( \frac{r_b(k_*)}{8} \right) + 2\xi_b^2(k_*) \right], \quad (4.8)$$

$$\begin{aligned} \kappa_T(k_*) - \kappa_S(k_*) &= \left( \frac{d^3 \ln r_b(k)}{d \ln k^3} \right)_* \\ &= \left[ 2 \left( \frac{r_b(k_*)}{8} \right)^3 - \frac{86}{9} \left( \frac{r_b(k_*)}{8} \right)^2 + \frac{4}{3} (6\xi_b^2(k_*) + 5\eta_b^2(k_*)) \left( \frac{r_b(k_*)}{8} \right) + 2\eta_b(k_*)\xi_b^2(k_*) + 2\sigma_b^3(k_*) \right] \end{aligned} \quad (4.9)$$

Here Eq (4.7-4.9) represent the running, running of the running and running of the double running of tensor-to-scalar ratio.

### B. Computation of Momentum integral

Now let us explicitly compute left hand side of the Eq (3.4). To serve this purpose I start with the computation of momentum integration where I investigate the possibility of four physical situations as mentioned in Eq (3.3) finally leading to:

$$\int_{k_e}^{k_{cmb}} d \ln k \sqrt{r_b(k)} = \begin{cases} \sqrt{r_b(k_*)} \ln \left( \frac{k_{cmb}}{k_e} \right) & \text{for Case I} \\ \frac{2\sqrt{r_b(k_*)}}{n_T(k_*) - n_S(k_*) + 1} \left[ \left( \frac{k_{cmb}}{k_*} \right)^{\frac{n_T(k_*) - n_S(k_*) + 1}{2}} - \left( \frac{k_e}{k_*} \right)^{\frac{n_T(k_*) - n_S(k_*) + 1}{2}} \right] & \text{for Case II} \\ \sqrt{r_b(k_*)} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{2(\alpha_T(k_*) - \alpha_S(k_*))}} \sqrt{\frac{2\pi}{(\alpha_T(k_*) - \alpha_S(k_*))}} \left[ \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} + \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \ln \left( \frac{k_{cmb}}{k_*} \right) \right) - \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} + \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \ln \left( \frac{k_e}{k_*} \right) \right) \right] & \text{for Case III} \\ \sqrt{r_b(k_*)} \left[ \left( \frac{3}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \left\{ \frac{k_{cmb}}{k_*} - \frac{k_e}{k_*} \right\} - \left( \frac{1}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \left\{ \frac{k_{cmb}}{k_*} \ln \left( \frac{k_{cmb}}{k_*} \right) - \frac{k_e}{k_*} \ln \left( \frac{k_e}{k_*} \right) \right\} + \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) \left\{ \frac{k_{cmb}}{k_*} \ln^2 \left( \frac{k_{cmb}}{k_*} \right) - \frac{k_e}{k_*} \ln^2 \left( \frac{k_e}{k_*} \right) \right\} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} \left\{ \frac{k_{cmb}}{k_*} \ln^3 \left( \frac{k_{cmb}}{k_*} \right) - \frac{k_e}{k_*} \ln^3 \left( \frac{k_e}{k_*} \right) \right\} \right] & \text{for Case IV.} \end{cases} \quad (4.10)$$

where in a realistic physical situation one assumes the pivot scale of momentum  $k_* \approx k_{cmb}$ . Now further substituting Eq (2.38) on Eq (4.10) I get:

$$\int_{k_e}^{k_{cmb}} d \ln k \sqrt{r_b(k)} = \begin{cases} \sqrt{r_b(k_*)} \Delta N_b & \text{for Case I} \\ \frac{2\sqrt{r_b(k_*)}}{n_T(k_*) - n_S(k_*) + 1} \left[ 1 - e^{-\Delta N_b \left( \frac{n_T(k_*) - n_S(k_*) + 1}{2} \right)} \right] & \text{for Case II} \\ \sqrt{r_b(k_*)} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{2(\alpha_T(k_*) - \alpha_S(k_*))}} \sqrt{\frac{2\pi}{(\alpha_T(k_*) - \alpha_S(k_*))}} \left[ \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} \right) - \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} - \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \Delta N_b \right) \right] & \text{for Case III} \\ \sqrt{r_b(k_*)} \left[ \left( \frac{3}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \{1 - e^{-\Delta N_b}\} - \left( \frac{1}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \right) \Delta N_b e^{-\Delta N_b} + \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) (\Delta N_b)^2 e^{-\Delta N_b} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^3 e^{-\Delta N_b} \right] & \text{for Case IV.} \end{cases} \quad (4.11)$$



Now for completeness let me concentrate on a limiting situation where  $\Delta N_b$  is small but within the observable range. In such a situation one has the following results:

$$\lim_{\Delta N_b \rightarrow \text{small}} \left[ \int_{k_e}^{k_{cmb}} d \ln k \sqrt{r_b(k)} \right] = \begin{cases} \sqrt{r_b(k_*)} \Delta N_b & \text{for Case II} \\ \sqrt{r_b(k_*)} \Delta N_b e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{2(\alpha_T(k_*) - \alpha_S(k_*))}} & \text{for Case III} \\ \sqrt{r_b(k_*)} \Delta N_b \left[ 1 - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) \Delta N_b \right. \\ \quad \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^2 \right] & \text{for Case IV.} \end{cases} \quad (4.12)$$

### C. Computation of Potential dependent integral

Next I compute the right hand side of the Eq (3.4). To serve this purpose I start with Eq (2.12).

$$\int_{\phi_e}^{\phi_{cmb}} d\phi \sqrt{V(\phi)} = \sqrt{V(\phi_0)} \left( \frac{\Delta\phi}{M_p} \right) \left[ 1 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{V'^n(\phi_0) M_p^n}{n! V(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^n \right] \approx \sqrt{V(\phi_0)} \left( \frac{\Delta\phi}{M_p} \right) \quad (4.13)$$

where in the next to last step I have used the convergent criteria of the series sum as mentioned earlier in this paper. Similarly from Eq (2.17) I get:

$$\int_{\phi_e}^{\phi_{cmb}} d\phi \sqrt{V(\phi)} = \sqrt{V_0} \left( \frac{\Delta\phi}{M_p} \right) \left[ 1 + \frac{1}{2} \sum_{m=1}^{\infty} \frac{C_{2m} M_p^{2m}}{(2m+1) V_0} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{2m} \right] \approx \sqrt{V_0} \left( \frac{\Delta\phi}{M_p} \right) \quad (4.14)$$

Now further clubbing Eq (4.13) and Eq (4.14) with/without  $\mathbf{Z}_2$  symmetric physical situation I get:

$$\int_{\phi_e}^{\phi_{cmb}} d\phi \sqrt{V(\phi)} \approx \sqrt{\mathcal{V}_{inf}} \left( \frac{\Delta\phi}{M_p} \right). \quad (4.15)$$

where the scale of inflation is determined by the symbol,  $\mathcal{V}_{inf} = V_0$  for  $\phi_0 = 0$  and  $\mathcal{V}_{inf} = V(\phi_0)$  for  $\phi_0 \neq 0$ .

### D. Relationship between brane tension and the scale of inflation in RS setup

Further I get the following relationship between brane tension and the scale of inflation:

$$\begin{aligned}
 & \text{Without } \mathbf{Z}_2 : \\
 & \frac{\sigma}{\mathcal{V}_{inf}} = \frac{1}{3} \times \left\{ \begin{aligned} & 1 && \text{for Case I} \\ & \left| \frac{4}{(n_T(k_*) - n_S(k_*) + 1)^2 |\Delta N_b|^2} \left| 1 - e^{-\Delta N_b \left( \frac{n_T(k_*) - n_S(k_*) + 1}{2} \right)} \right|^2 \right. && \text{for Case II} \\ & \left. \frac{1}{|\Delta N_b|^2} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{(\alpha_T(k_*) - \alpha_S(k_*))}} \left( \frac{2\pi}{(\alpha_T(k_*) - \alpha_S(k_*))} \right) \right. \\ & \left. \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} \right) \right. \\ & \quad - \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} - \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \Delta N_b \right) \Big|^2 && \text{for Case III} \\ & \frac{1}{|\Delta N_b|^2} \left| \left( \frac{3}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} \right. \right. \\ & \quad - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \Big) \left\{ 1 - e^{-\Delta N_b} \right\} - \left( \frac{1}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} \right. \\ & \quad + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \Big) \Delta N_b e^{-\Delta N_b} \\ & \quad - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) (\Delta N_b)^2 e^{-\Delta N_b} \\ & \quad \left. \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^3 e^{-\Delta N_b} \right|^2 \right. && \text{for Case IV.} \end{aligned} \right. \quad (4.16)
 \end{aligned}$$

With  $\mathbf{Z}_2$  :

$$\begin{aligned}
 & \frac{\sigma}{\mathcal{V}_{inf}} = \frac{r(k_*)}{48 \left( n_S(k_*) - 1 + \frac{3r(k_*)}{8} \right)^2} \times \left\{ \begin{aligned} & 1 && \text{for Case I} \\ & \left| \frac{4}{(n_T(k_*) - n_S(k_*) + 1)^2 |\Delta N_b|^2} \left| 1 - e^{-\Delta N_b \left( \frac{n_T(k_*) - n_S(k_*) + 1}{2} \right)} \right|^2 \right. && \text{for Case II} \\ & \left. \frac{1}{|\Delta N_b|^2} e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{(\alpha_T(k_*) - \alpha_S(k_*))}} \left( \frac{2\pi}{(\alpha_T(k_*) - \alpha_S(k_*))} \right) \right. \\ & \left. \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} \right) \right. \\ & \quad - \operatorname{erfi} \left( \frac{n_T(k_*) - n_S(k_*) + 1}{\sqrt{2(\alpha_T(k_*) - \alpha_S(k_*))}} - \sqrt{\frac{(\alpha_T(k_*) - \alpha_S(k_*))}{8}} \Delta N_b \right) \Big|^2 && \text{for Case III} \\ & \frac{1}{|\Delta N_b|^2} \left| \left( \frac{3}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} \right. \right. \\ & \quad - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \Big) \left\{ 1 - e^{-\Delta N_b} \right\} - \left( \frac{1}{2} - \frac{n_T(k_*) - n_S(k_*)}{2} \right. \\ & \quad + \frac{\alpha_T(k_*) - \alpha_S(k_*)}{8} - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{24} \Big) \Delta N_b e^{-\Delta N_b} \\ & \quad - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) (\Delta N_b)^2 e^{-\Delta N_b} \\ & \quad \left. \left. - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^3 e^{-\Delta N_b} \right|^2 \right. && \text{for Case IV.} \end{aligned} \right. \quad (4.17)
 \end{aligned}$$

In the limiting situation when  $\Delta N_b$  is small but lies within the observable window, I get the following relationship between brane tension and the scale of inflation:

Without  $Z_2$  :

$$\lim_{\Delta N_b \rightarrow \text{small}} \frac{\sigma}{V_{inf}} = \frac{1}{3} \times \begin{cases} 1 & \text{for Case II} \\ e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{(\alpha_T(k_*) - \alpha_S(k_*))}} & \text{for Case III} \\ \left| 1 - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) \Delta N_b - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^2 \right|^2 & \text{for Case IV.} \end{cases} \quad (4.18)$$

With  $Z_2$  :

$$\lim_{\Delta N_b \rightarrow \text{small}} \frac{\sigma}{V_{inf}} = \frac{r(k_*)}{48 \left( n_S(k_*) - 1 + \frac{3r(k_*)}{8} \right)^2} \times \begin{cases} 1 & \text{for Case II} \\ e^{-\frac{(n_T(k_*) - n_S(k_*) + 1)^2}{(\alpha_T(k_*) - \alpha_S(k_*))}} & \text{for Case III} \\ \left| 1 - \left( \frac{\kappa_T(k_*) - \kappa_S(k_*)}{48} - \frac{\alpha_T(k_*) - \alpha_S(k_*)}{16} \right) \Delta N_b - \frac{\kappa_T(k_*) - \kappa_S(k_*)}{144} (\Delta N_b)^2 \right|^2 & \text{for Case IV.} \end{cases} \quad (4.19)$$

### E. Computation of analytic expression for $\Delta N_b$ in terms of potential

Let me now compute the analytical expression for  $\Delta N_b$  using Eq (2.27) and the explicit form of the potential stated in Eq (2.12) and Eq (2.17) for consistency check.

Without  $Z_2$  :

$$\begin{aligned} \Delta N_b \approx & \frac{V^2(\phi_0)\Delta\phi}{2\sigma V'(\phi_0)M_p^2} \left[ 1 - \sum_{p=1}^{\infty} \frac{V'^p(\phi_0)M_p^{p-1}}{(p-1)!V'(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{p-1} + 2 \sum_{n=1}^{\infty} \frac{V'^n(\phi_0)M_p^n}{n!V(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^n \right. \\ & + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{V'^n(\phi_0)V'^m(\phi_0)M_p^{m+n}}{n!m!V^2(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{n+m} + 2 \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} \frac{V'^n(\phi_0)V'^p(\phi_0)M_p^{n+p-1}}{n!(p-1)!V(\phi_0)V'(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{n+p-1} \\ & \left. + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} \frac{V'^n(\phi_0)V'^m(\phi_0)V'^p(\phi_0)M_p^{n+m+p-1}}{n!m!(p-1)!V^2(\phi_0)V'(\phi_0)} \left( \frac{\phi_e - \phi_0}{M_p} \right)^{n+m+p-1} \right] \approx \frac{V^2(\phi_0)\Delta\phi}{2\sigma V'(\phi_0)M_p^2}, \end{aligned} \quad (4.20)$$

With  $Z_2$  :

$$\begin{aligned} \Delta N_b \approx & \frac{V_0^2\Delta\phi}{4\sigma m^2 M_p^2} \left[ \frac{1}{\Delta\phi} \ln \left( 1 + \frac{\Delta\phi}{\phi_e} \right) + 2 \sum_{m=1}^{\infty} \frac{C_{2m}M_p^{2m}}{V_0} \left( \frac{\phi_e}{M_p} \right)^{2m} - \sum_{p=2}^{\infty} \frac{pC_{2p}M_p^{2p-2}}{m^2} \left( \frac{\phi_e}{M_p} \right)^{2p-2} \right. \\ & + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{C_{2m}C_{2n}M_p^{2(n+m)}}{V_0^2} \left( \frac{\phi_e}{M_p} \right)^{2(n+m)} - 2 \sum_{m=1}^{\infty} \sum_{p=2}^{\infty} \frac{pC_{2p}C_{2m}M_p^{2(p+m)-2}}{V_0m^2} \left( \frac{\phi_e}{M_p} \right)^{2(p+m)-2} \\ & \left. - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{p=2}^{\infty} \frac{pC_{2p}C_{2m}C_{2n}M_p^{2(p+m+n)-2}}{V_0^2m^2} \left( \frac{\phi_e}{M_p} \right)^{2(p+m+n)-2} \right] \approx \frac{V_0^2\Delta\phi}{4\sigma m^2 M_p^2 \phi_e}, \end{aligned} \quad (4.21)$$

where for both the cases convergence criteria of the series sum are imposed.

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